The Peak of the Stellar IMF from Supersonic Turbulence

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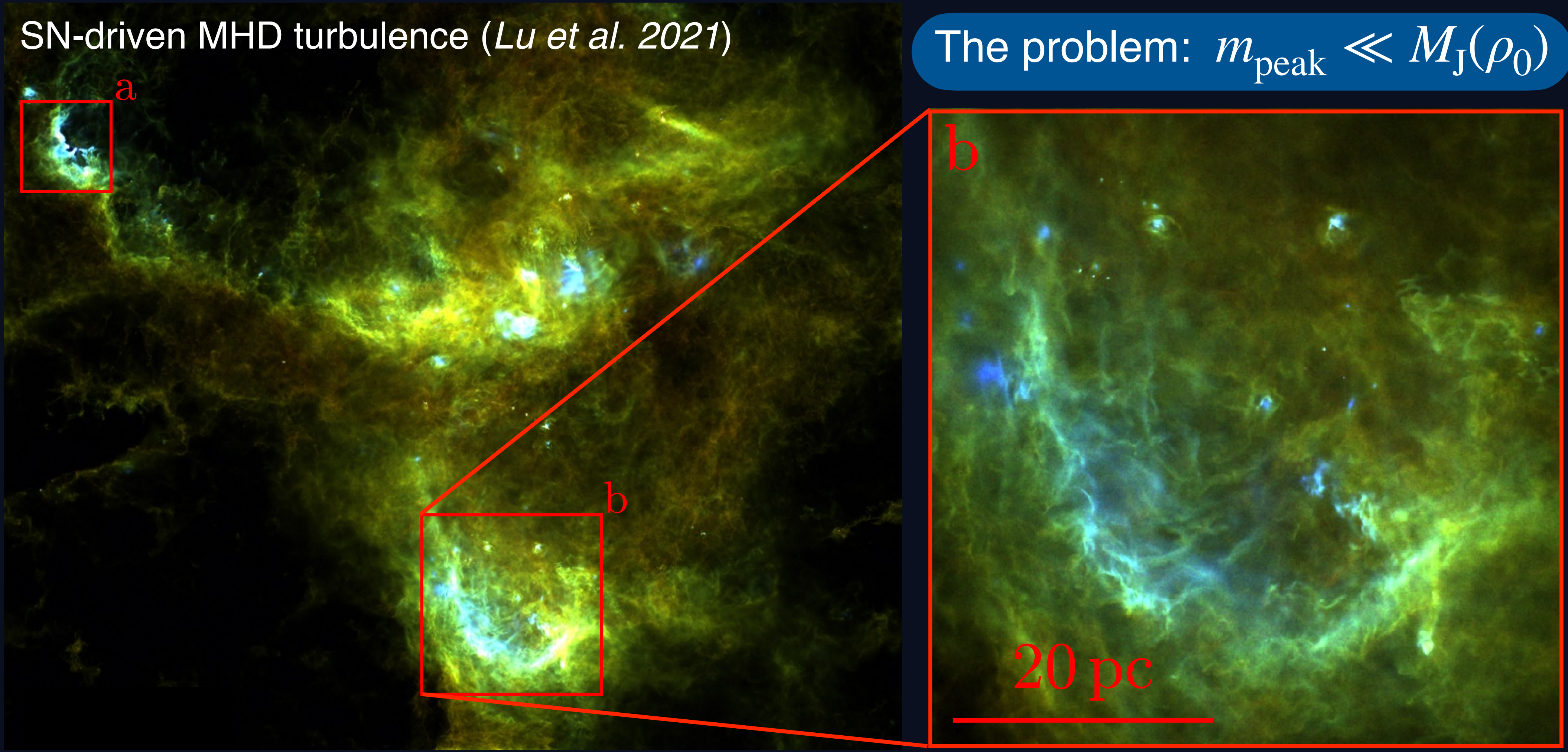
Haugbølle et al. 2018, ApJ, 854, 35
Padoan et al. 2026 (in preparation)

TURBULENT FRAGMENTATION



The problem: $m_{\rm peak} \ll M_{\rm J}(\rho_0)$

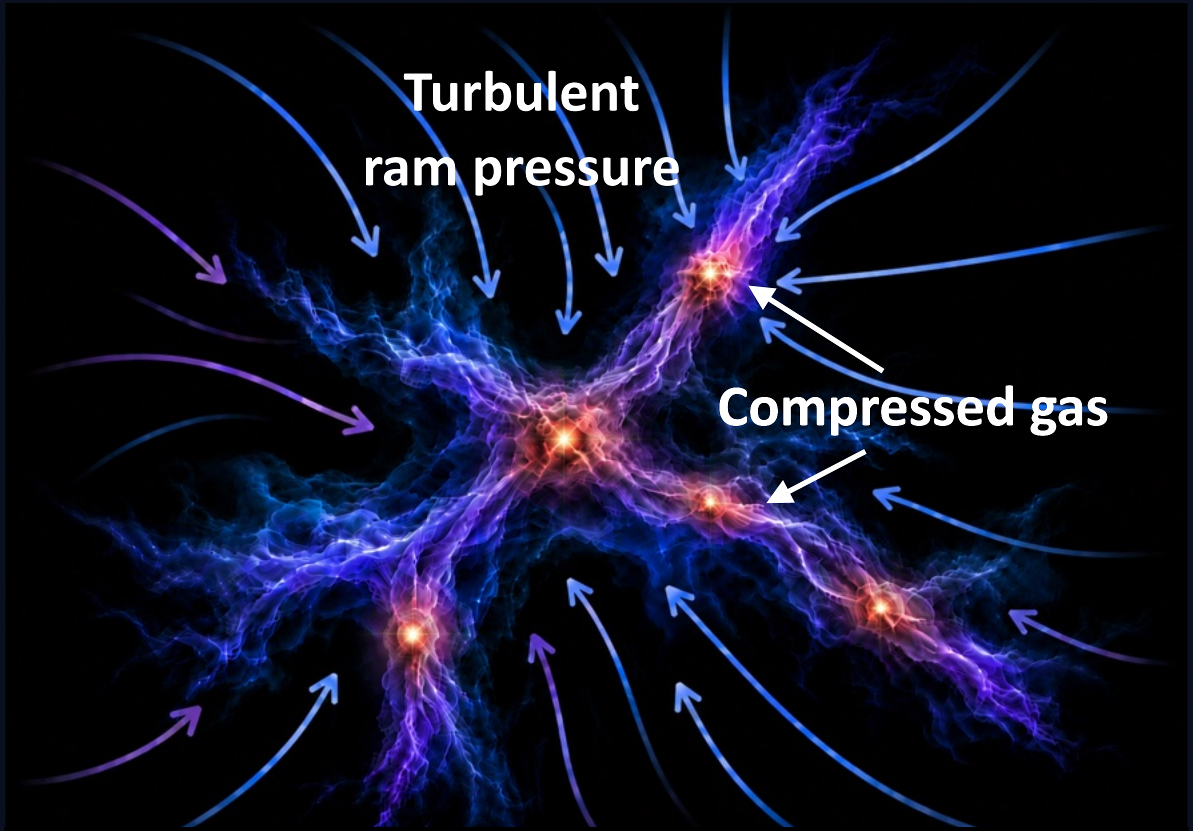
Shocks, filaments and cores:

Broad density and pressure distributions can explain $m_{\rm peak} \ll M_{\rm J}(\rho_0)$.

A ZEROTH-ORDER PICTURE: The Turbulent Bonnor-Ebert Mass

$$M_{\rm BE,turb} \equiv \frac{1.18\, c_s^4}{G^{3/2} P_{\rm turb}^{1/2}} \simeq \frac{M_{\rm BE,0}}{\mathscr{M}_s}$$

$$P_{\rm turb} \simeq \rho_0 v_{\rm rms}^2 = \rho_0 \left(\mathscr{M}_s c_s\right)^2$$



Haugbølle, Padoan, and Nordlund (2018)

The Ansatz: $m_{\rm peak} \equiv \epsilon\, M_{\rm BE,turb}$

Larson scaling ($\alpha_{\rm vir} \simeq 1.1$):

$$M_{\rm BE,0} \simeq 0.4\, \mathscr{M}_s\, M_\odot$$

$$\Rightarrow \quad M_{\rm BE,turb} \simeq 0.4\, M_\odot$$

$$\Rightarrow \quad m_{\rm peak} \simeq \epsilon\, M_{\rm BE,turb} \approx 0.2\, M_\odot$$

Universal peak, consistent with Chabrier's IMF

DIFFERENT MODELS OF TURBULENT FRAGMENTATION

In all models, turbulence creates density fluctuations —> density PDF

They differ in what sets the characteristic mass:

Turbulent Compression	Turbulent Support	Hierarchical Fragmentation
Thermal support in compressed gas	**Thermal and turbulent support**	**Smallest self-gravitating fragment**
Bonnor-Ebert mass	**Turbulent Jeans mass**	**Sonic Mass**
Padoan & Nordlund 2002 (PN02)	*Hennebelle & Chabrier 2008 (HC08)*	*Hopkins 2012 (H12)*

PREDICTED MACH DEPENDENCE

We normalize $m_{\rm peak}$ by $M_{\rm BE,0}$ to remove the mean-density thermal mass scale:

Turbulent Compression

$$\frac{m_{\rm peak}}{M_{\rm BE,0}} \propto \mathscr{M}^{-1.2}$$

Turbulent Support

$$\frac{m_{\rm peak}}{M_{\rm BE,0}} \propto \mathscr{M}^{-1.5}$$

Hierarchical Fragmentation

$$\frac{m_{\rm peak}}{M_{\rm BE,0}} \propto \mathscr{M}^{-4.3}$$

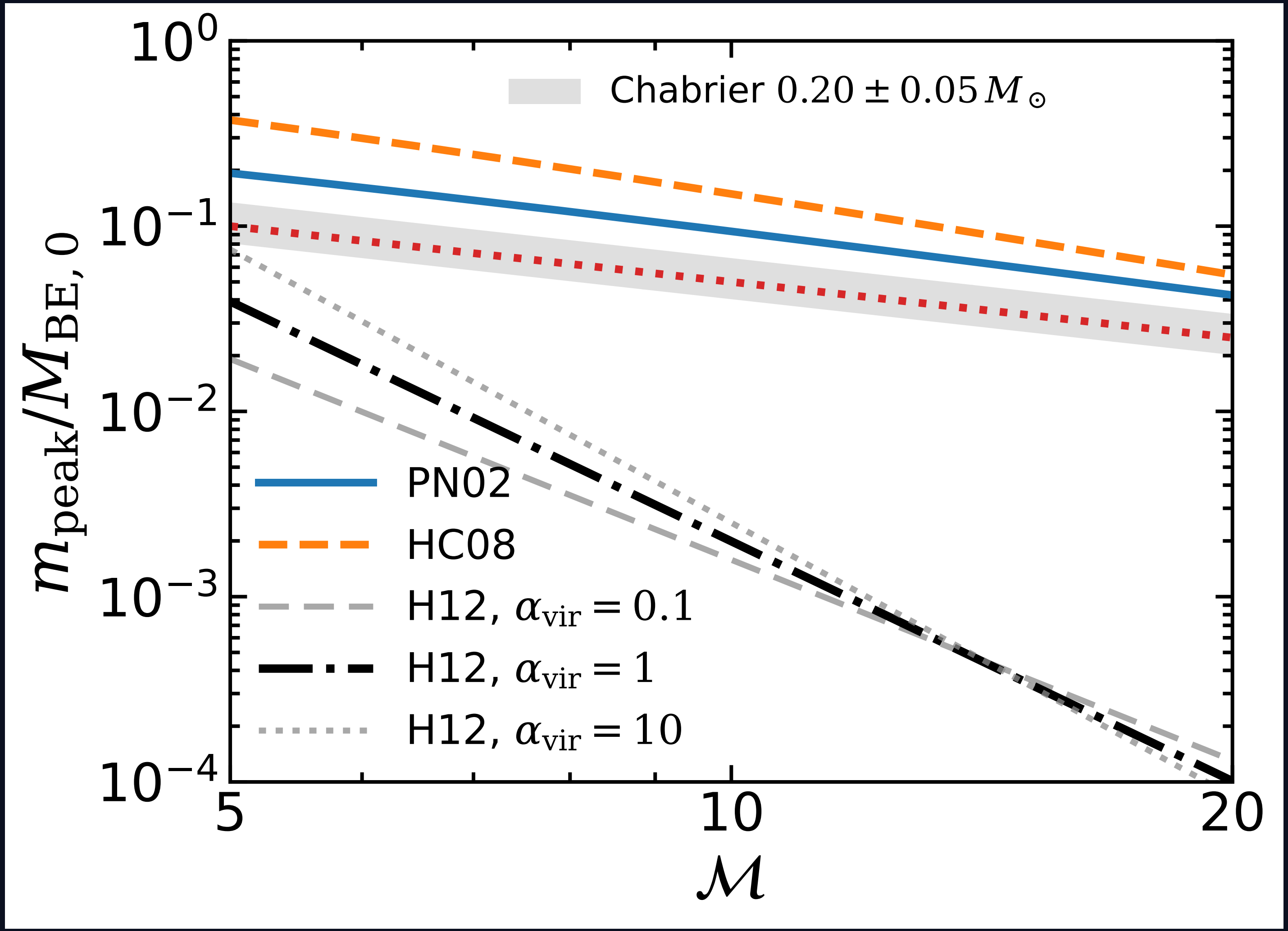
Why is Hopkins (2012) so steep?

Same support picture as HC08, but cores are the smallest self-gravitating fragments:

SMALLEST FRAGMENT —> SONIC SCALE —> $m_{\rm peak}/M_{\rm BE,0} = f(\mathscr{M}, \alpha_{\rm vir}, L_{\rm out})$

H12 is much steeper and not a universal function of $\mathscr{M}$ alone.

PREDICTED MACH DEPENDENCE



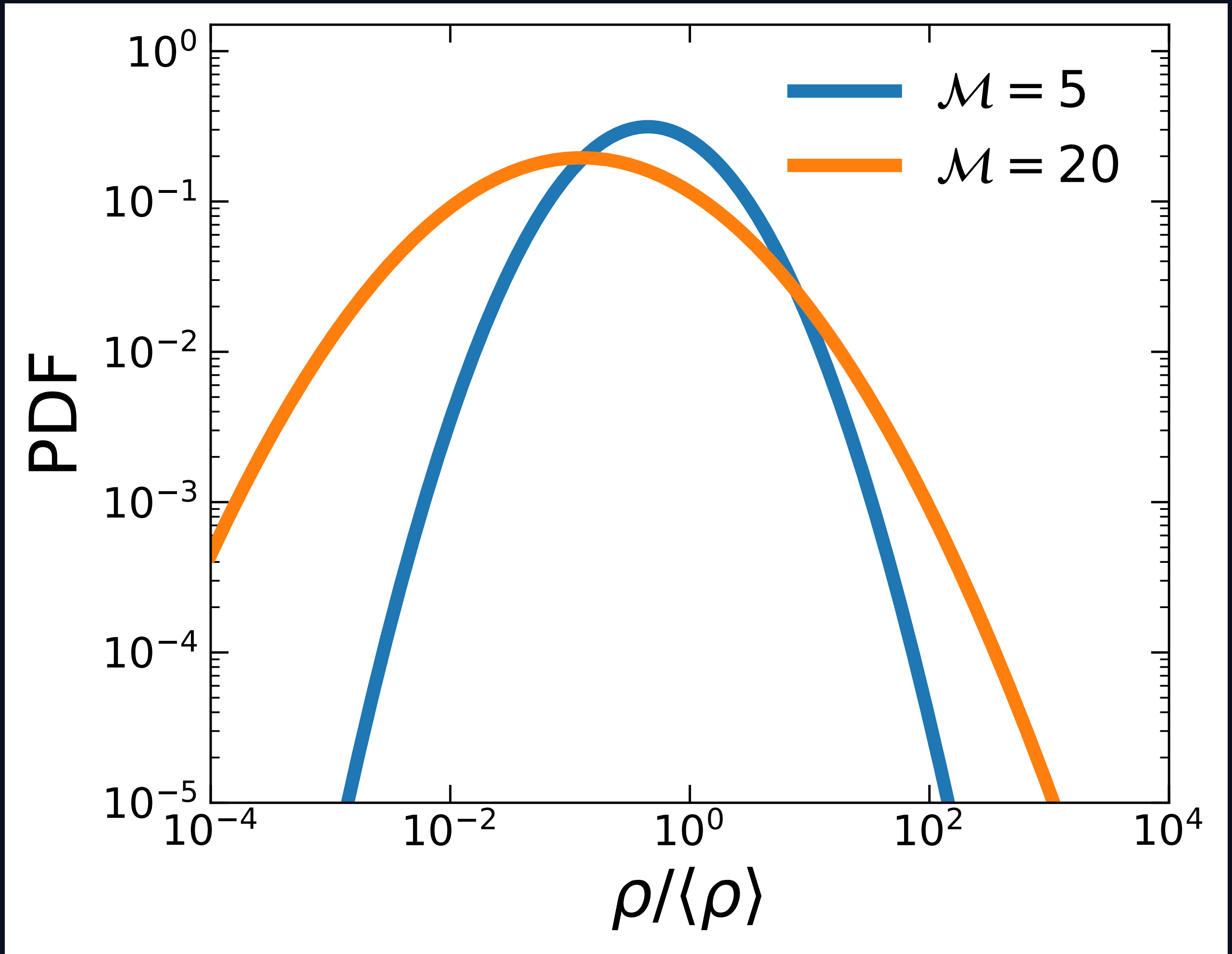
Different normalizations as well:

- HC08 slightly above PN02 due to inclusion of turbulent support
- H12 much lower due to small sonic mass in typical clouds ($\mathscr{M} \sim 10$ at few pc and $\alpha_{\rm vir} \sim 1$).

No feedbacks in the models!

THE KEY PREDICTION OF TURBULENT FRAGMENTATION

$$\mathscr{M}\uparrow \underset{\checkmark}{\Longrightarrow} P(\rho)\ \text{broadens} \underset{=}{\Longrightarrow} m_{\text{peak}}\downarrow$$



✓ Density-PDF dependence: extensively tested

= ***IMF-peak dependence: not directly tested***

WHY HAS THIS NEVER BEEN TESTED?

Testing $m_{\text{peak}}(\mathscr{M})$ requires a **converged IMF peak**

Physical requirements

- widely separated Mach numbers
- multiple realizations
- non-ideal MHD at AU scales

Convergence requirements

- refinement level, l_{max}
- sink-creation density, ρ_{s}

3 AU + ambipolar diffusion —> $\sim 10^8$ local time steps per Myr

Now feasible with **DISPATCH**: asynchronous, task-based, *local dt*

THE NUMERICAL TEST — Isolate turbulence first

ONE RUN

- Periodic box
- L_box = 4 pc

- isothermal
- randomly driven
- self-gravity + sinks
- MHD + ambipolar drift

DISPATCH code:
AMR + local dt

PARAMETER GRID

Main scan:

$$\mathcal{M}_s = 5, 10, 20$$

$$\alpha_{vir} = 0.1 - 4.5$$

$\mathcal{M}_A \sim \mathcal{M}_s/2$ (limited tests)

CONVERGENCE

$$l_{max} = 12 - 18$$

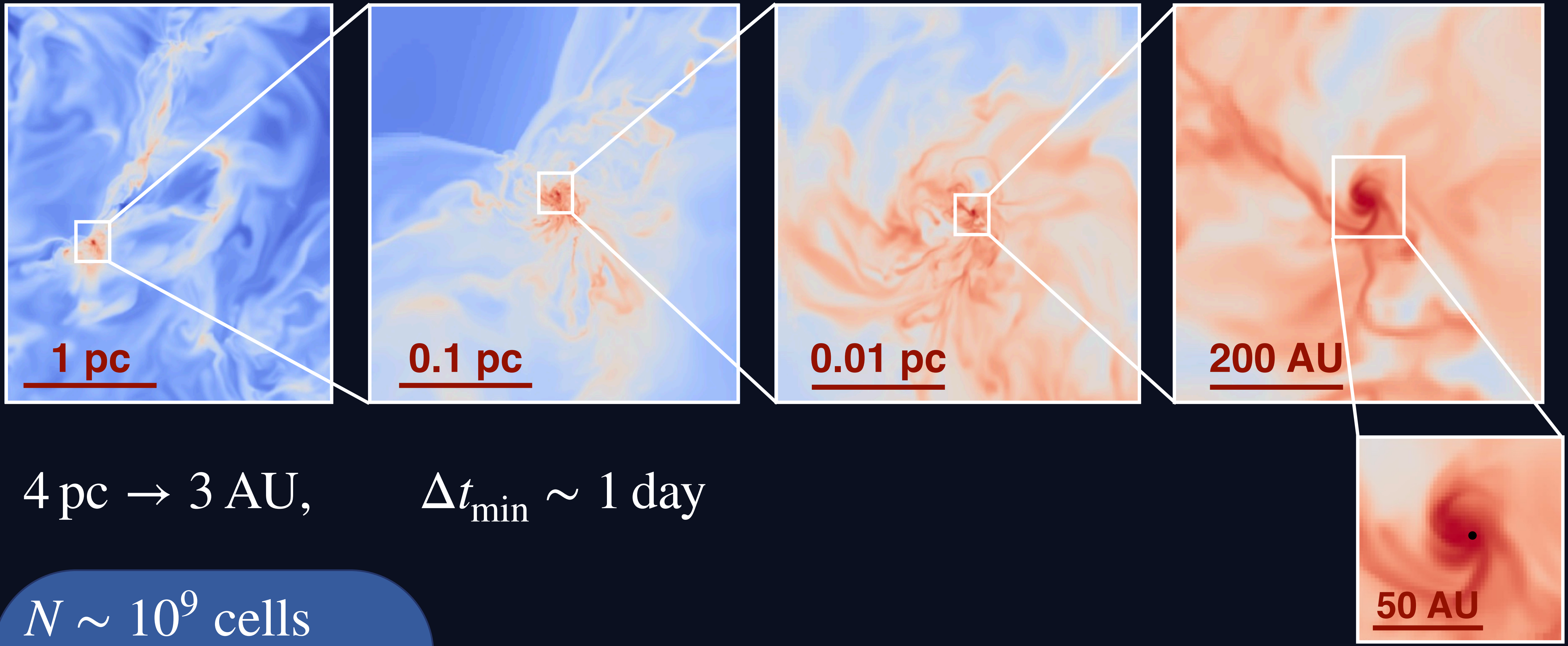
$$\Delta x_{min} = 195 - 3\ \mathrm{AU}$$

- sink-creation density
- multiple realizations
- Jeans resolution

No feedback — by design

A converged **turbulence-only** baseline before adding feedback

FROM TURBULENT CLOUDS TO AU-SCALE ACCRETION



$4\,\mathrm{pc} \rightarrow 3\,\mathrm{AU}$, $\Delta t_{\mathrm{min}} \sim 1\,\mathrm{day}$

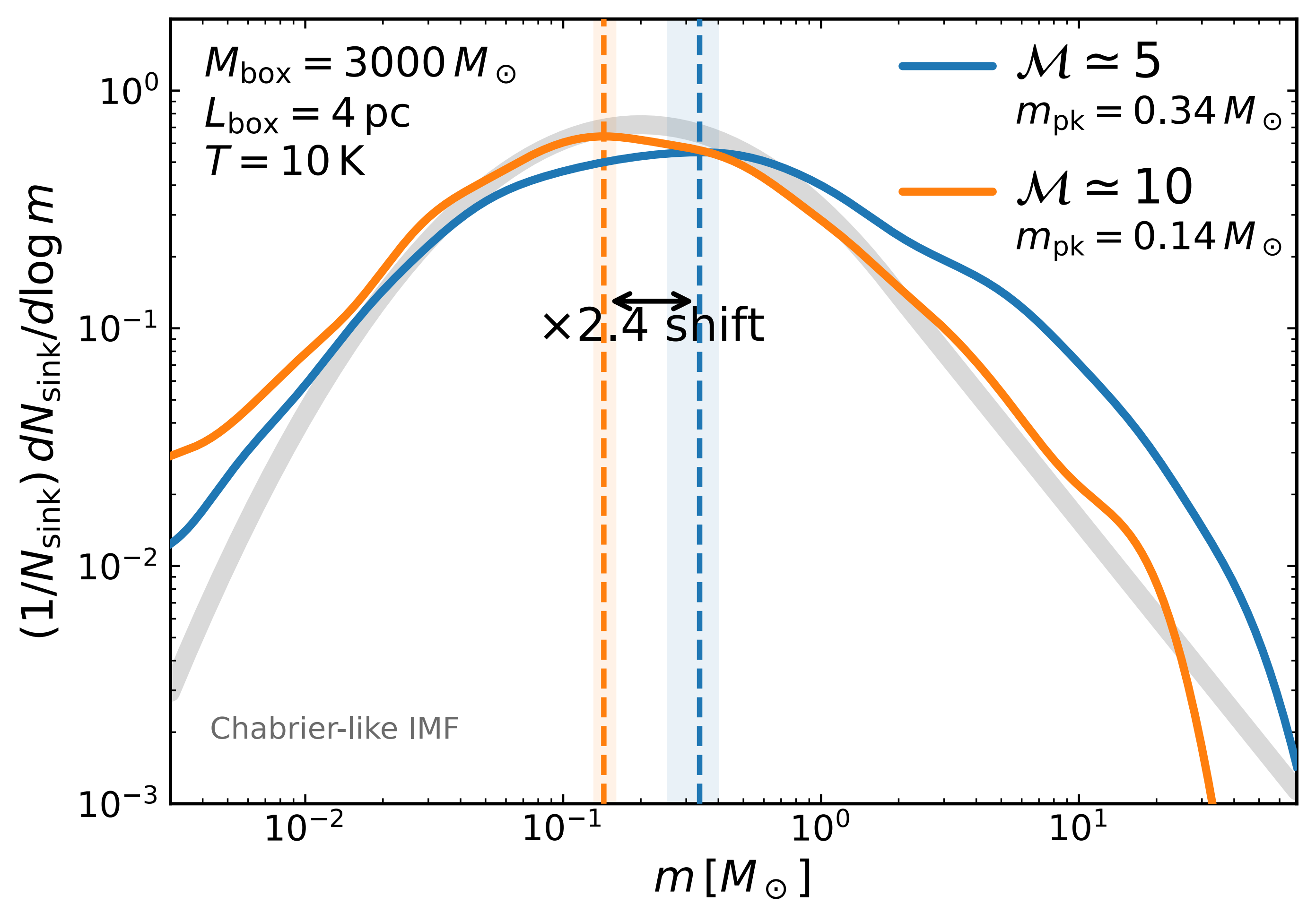
$N \sim 10^9$ cells

$\Delta m_{\mathrm{min}} \sim 10^{-6}\,M_\odot$

—> *STARFORGE x 100*

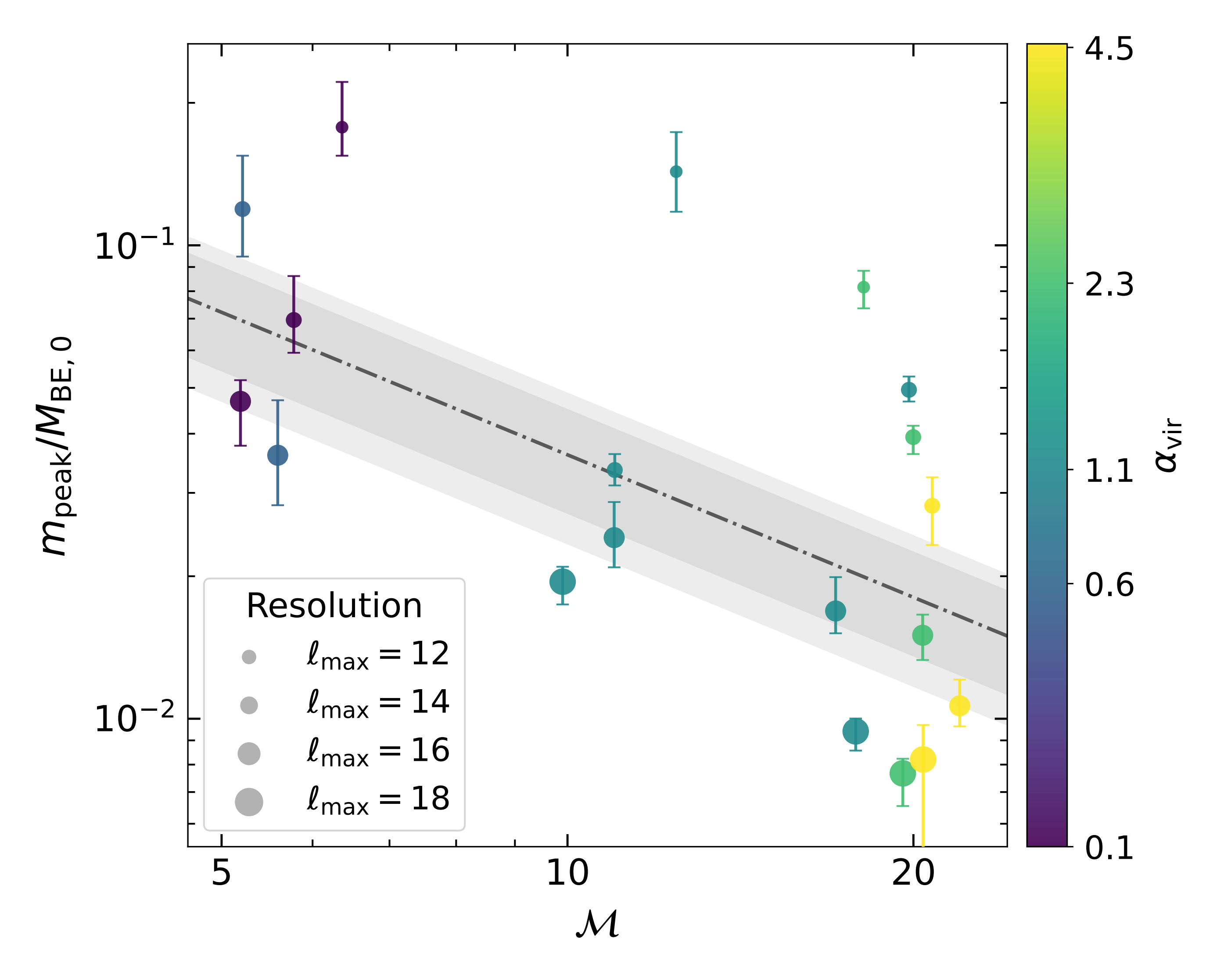
$\mathscr{M} \sim 5$, $l_{\mathrm{max}} = 18$

THE IMF PEAK MOVES WITH MACH NUMBER



$$\mathscr{M} = 5 \rightarrow 10 \quad \Rightarrow \quad m_{\rm pk} = 0.34 \rightarrow 0.14\,M_\odot$$

PEAK MASS FROM THE SIMULATION GRID



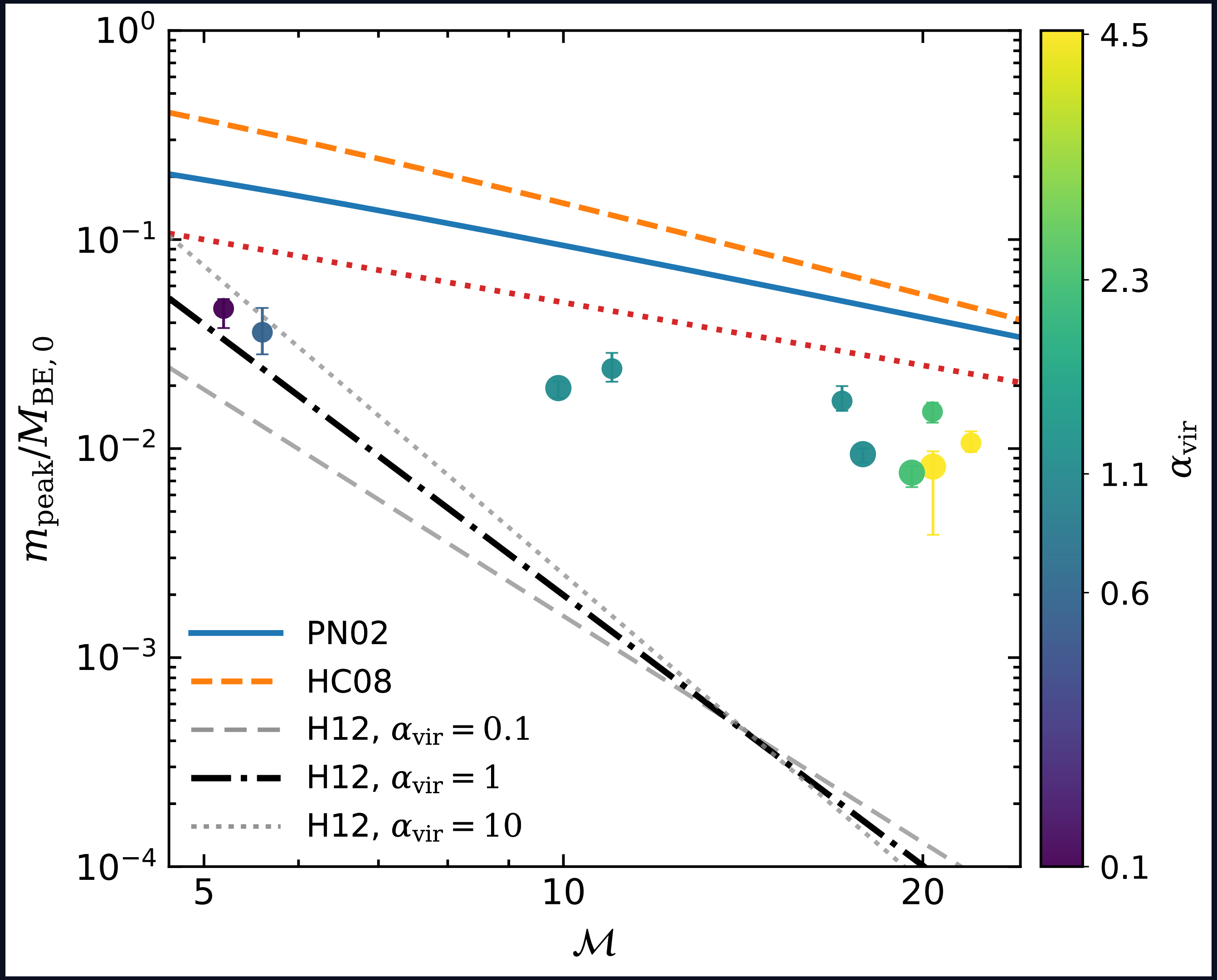
- points: run families
- size: numerical resolution
- color: α_{vir}
- shaded band: Chabrier peak + Larson relations

Consistent with a universal

$$m_{peak}/M_{BE,0} \simeq \epsilon/\mathscr{M}$$

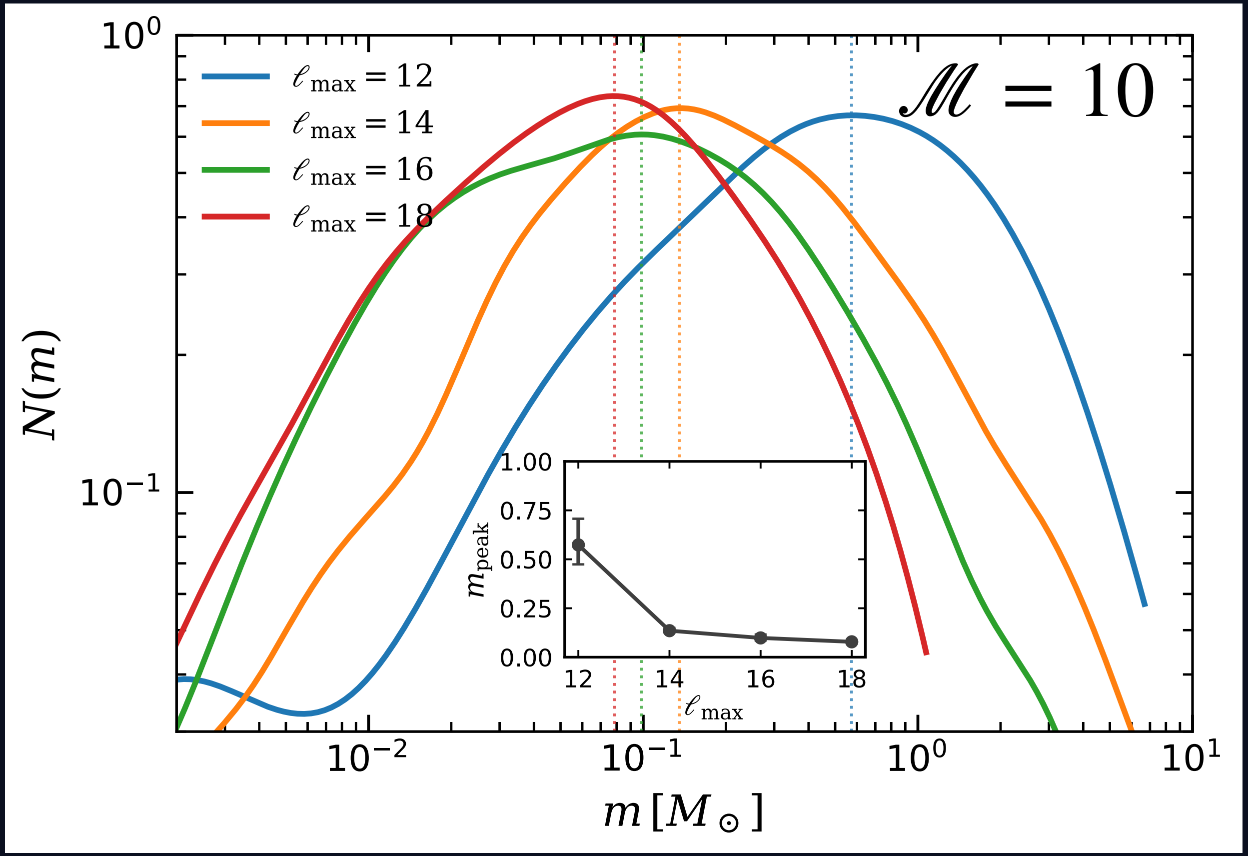
despite a factor 45 range in α_{vir}

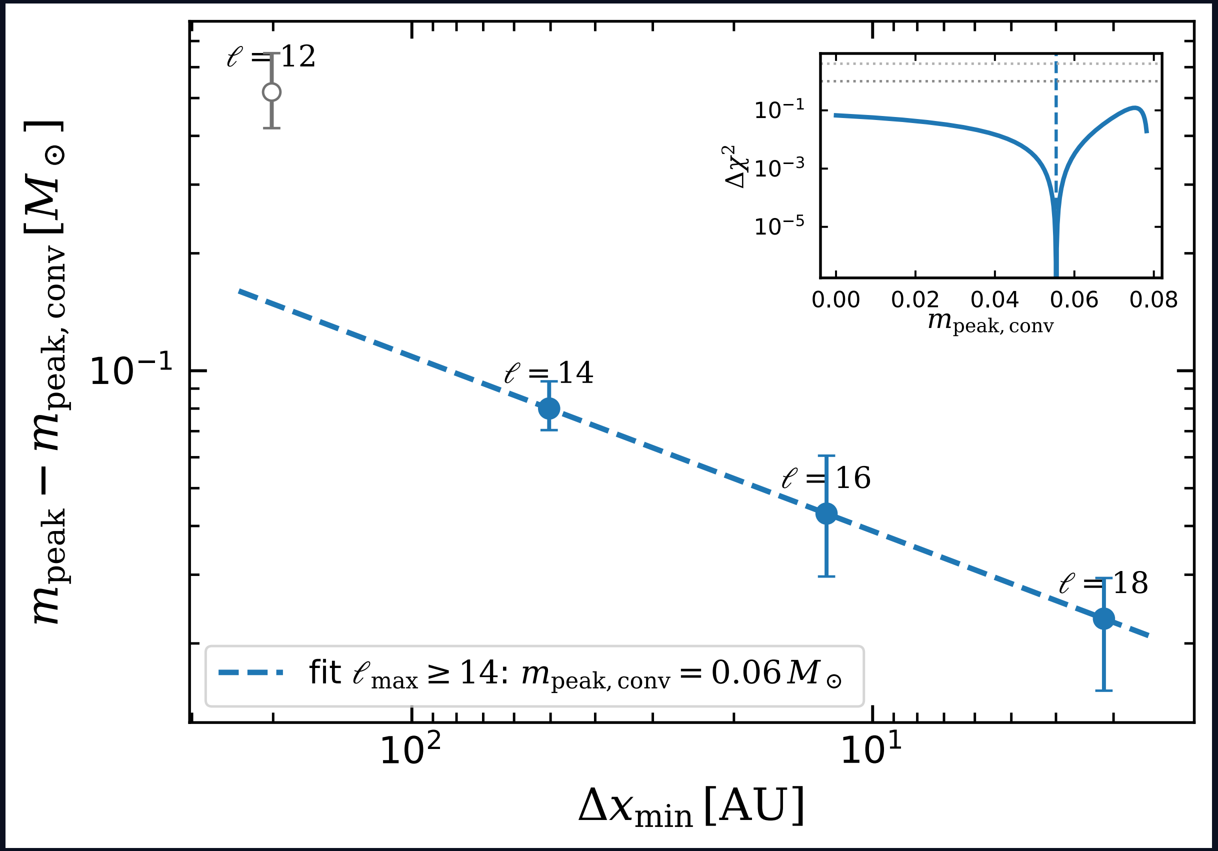
SIMULATIONS VERSUS MODELS (3–12 AU resolution)



- PN02 and H18 give the closest slope and normalization.
- HC08 lies systematically high and is slightly too steep.
- H12 is much too steep, and too low by ~2 dex at $\mathscr{M} = 20$.
- The Mach trend predicted by turbulent fragmentation is strongly supported.
- The sonic-scale H12 prediction is not supported by these simulations.

NUMERICAL CONVERGENCE





Preliminary convergence at $\mathscr{M} = 10 - 20$

CONCLUSIONS

1. The simulations confirm the key prediction:

$$\mathscr{M}\uparrow \implies m_{\rm peak}/M_{\rm BE,0}\downarrow$$

2. The trend is consistent with:

$$m_{\rm peak}/M_{\rm BE,0} \simeq \epsilon/\mathscr{M}$$

3. No clear residual dependence on $\alpha_{\rm vir}$ over a factor of 45.

4. PN02/H18 are closest; HC08 is somewhat high/steep; H12 is much too steep.

(similar conclusions for IMF shape —> *Nam, Federrath and Krumholz 2020*)

5. Convergence requires AU-scale, non-ideal MHD.

Feedback comes next — after the turbulence-only IMF is converged.